

The angular size - redshift relation in power-law cosmologies

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A linear evolution of the cosmological scale factor is a feature in several models designed to solve the cosmological constant problem via a coupling between scalar or tensor classical fields to the space-time curvature as well as in some alternative gravity theories. In this paper, by assuming a general time dependence of the scale factor, $R \sim t^\alpha$, we investigate observational constraints on the dimensionless parameter α from measurements of the angular size for a large sample of milliarcsecond compact radio sources. In particular, we find that a strictly linear evolution, i.e., $\alpha \simeq 1$ is favoured by these data, which is also in agreement with limits obtained from other independent cosmological tests. The dependence of the critical redshift z_m (at which a given angular size takes its minimal value) with the index α is briefly discussed.

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I. INTRODUCTION

Over the past few years, an impressive convergence of observational facts have led cosmologists to search for alternative cosmologies. Among these facts, the most remarkable finding surely comes from distance measurements of type Ia supernovae (SNe Ia) that suggest that the expansion of the universe is speeding up, not slowing down [1]. Such a result, when combined with the latest Cosmic Microwave Background (CMB) data and clustering estimates, seems to provide a compelling evidence for a non-zero cosmological constant Λ [2].

On the other hand, it is also well known that the same welcome properties that make models with a relic cosmological constant (Λ CDM) our best description of the observed universe also result in a serious fine tuning problem [3]. The basic reason is the widespread belief that the early universe evolved through a cascade of phase transitions, thereby yielding a vacuum energy density which is presently 120 orders of magnitude smaller than its value at the Planck time. Such a discrepancy between theoretical expectations and empirical observations constitutes a fundamental problem at the interface uniting astrophysics, cosmology and particle physics. In the last years, several attempts have been done in order to alleviate the cosmological constant problem. For example, in the so-called dynamical $\Lambda(t)$ scenarios (or deflationary cosmology), the cosmological term is a function of time and its presently observed value is a remnant of

the primordial inflationary/deflationary stage [4]. Other examples are scenarios in which the evolution of classical fields are coupled to the curvature of the space-time background in such a way that their contribution to the energy density self-adjusts to cancel the vacuum energy [5], as well as some recent ideas of a SU(2) cosmological instanton dominated universe [6]. At least in the two latter examples, an interesting feature is a power-law growth for the cosmological scale factor $R(t) \sim t^\alpha$, where α is determined by observational data and takes values within the interval $0 \leq \alpha \leq \infty$. A linear evolution of the scale factor is also supported in some alternative gravity theories, e.g., non-minimally coupled scalar-tensorial theories [7], as well as in standard model with a specially chosen equation of state [8].

The motivation for seriously investigating these power-law scenarios comes from several considerations. For example, for $\alpha \geq 1$ such models do not suffer the horizon problem. Moreover, the scale factor in such theories does not constrain the matter density parameter and, therefore, they are free of the flatness problem. There are also observational motivations for considering power-law cosmologies. For $\alpha \geq 1$, the predicted age of the universe is $t_o \geq H_o^{-1}$, i.e., at least 30 – 50% greater than the prediction of the flat standard model (without cosmological constant), thereby making such a universe comfortably in agreement with the recent age estimates of globular clusters and high- z redshift galaxies [9]. Recently, it was shown that such models are also compatible with the currently data of SNe Ia for a power index $\alpha \simeq 1$ [10] (see, however, [11] for a discussion involving SNe Ia and primordial nucleosynthesis constraints).

In this paper we explore the prospects for constraining the power-law index α from the angular size measurements of high- z milliarcsecond radio sources. We also

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study the influence of this dimensionless parameter on the minimal redshift at which the angular size of an extragalactic source takes its minimal value. For the sake of simplicity and also motivated by the latest results of CMB analyses we focus our attention on flat scenarios. We show that a good agreement between theory and observation is possible if $\alpha = 1.0 \pm 0.3$ at 68% c.l. with the characteristic length of the sources of the order of $l \simeq 26$ pc (for $H_o = 72 \text{ km s}^{-1} \text{ Mpc}^{-1}$).

This paper is organized as follows. In section II some basic assumptions and distance formulas are presented. The dependence of the minimal redshift z_m on the index α is studied in section III. In section IV we analyse the constraints from angular size data on this class of cosmologies and compare them with other independent limits. In section V our main conclusions are presented.

II. POWER-LAW COSMOLOGIES: BASIC EQUATIONS

Let us now consider the flat Friedmann-Robertson-Walker (FRW) line element

$$ds^2 = c^2 dt^2 - R^2(t) [d\xi^2 + \xi^2(d\theta^2 + \sin^2\theta d\phi^2)], \quad (1)$$

where ξ , θ , and ϕ are dimensionless comoving coordinates and $R(t)$ is the cosmological scale factor.

We study a general class of power law cosmology in which the scale factor is given in terms of the arbitrary dimensionless parameter α

$$R(t) \propto \frac{c}{H_o} \left(\frac{t}{t_o} \right)^\alpha. \quad (2)$$

The expansion rate of the universe is described by the Hubble parameter, $H(t) = \dot{a}/a = \alpha/t$ while the *present* expansion rate is defined by a Hubble *constant*, equal in this model to

$$H_o = \alpha/t_o \quad (3)$$

(here and subsequently the subscript “o” refers to its present value). By comparing the above expression with some recent estimates of the age parameter, it is possible to obtain directly limits on the dimensionless parameter α . For example, by assuming $t_o = 13 \pm 2$ Gyr as a median value for the age estimates of globular clusters and using $H_o = 72 \pm 8 \text{ km s}^{-1} \text{ Mpc}^{-1}$, in accordance with the final results of the *Hubble Space Telescope* Key Project [12], we find $\alpha = 0.96 \pm 0.19$.

As usual, the scale factor and the redshift are related by $a/a_o = (1+z)^{-1}$ while the dimensionless Hubble parameter now takes the form

$$h(z) \equiv \frac{H(z)}{H_o} = (1+z)^{1/\alpha}. \quad (4)$$

From the above equations, it is straightforward to show that the comoving distance $\xi(z)$ for flat geometries is

given by

$$\xi(z) = \frac{c\alpha}{R_o H_o (1-\alpha)} \left[1 - (1+z)^{\frac{\alpha-1}{\alpha}} \right], \quad (5)$$

which, in the limit $\alpha \rightarrow 1$, reduces to

$$\xi(z) = \frac{c}{R_o H_o} \ln(1+z). \quad (6)$$

The angular size redshift relation for a rod of intrinsic length l is easily obtained by integrating the spatial part of Eq. (1) for ξ and ϕ fixed [13]. One finds

$$\theta(z) = \frac{l(1+z)}{R_o \xi(z)} = \frac{D(1+z)(1-\alpha)}{\alpha \left[1 - (1+z)^{\frac{\alpha-1}{\alpha}} \right]}, \quad (7)$$

where the characteristic length l is measured in parsecs (for compact radio sources) and the characteristic angular scale $D = lH_o/c$ is given in milliarcsecond (mas).

III. THE CRITICAL REDSHIFT

As is well known, the existence of a critical redshift z_m on the angular size - redshift relation may be qualitatively understood in terms of the cosmological expansion. The light observed today from a source at a given redshift z was emitted when the object was closer (for a detailed discussion see [14]). Although nearby objects are not affected, a fixed angular size of extragalactic sources at high- z is seen initially decreasing to a minimal value, say, z_m , and afterwards increasing to higher redshifts. Although this minimal redshift test cannot discriminate by itself among different cosmological models (different scenarios may predict the same z_m values) [15], a precise determination of z_m or, equivalently, the corresponding minimal angular size value $\theta(z_m)$, may constitute, when combined with other cosmological tests, a powerful tool to check the validity of realistic world models. Such an effect was first predicted by Hoyle [16], originally aiming at distinguishing the steady state and Einstein-de Sitter cosmologies. Later on, the accumulated evidence against the original version of the steady state model have put such a scenario aside, and more recently the same is occurring with the theoretically favoured critical density FRW model [17].

The redshift z_m at which the angular size takes its minimal value is the one cancelling out the derivative of θ with respect to z . From Eq. (7), we find

$$z_m = \alpha^{\frac{\alpha}{\alpha-1}} - 1, \quad (8)$$

which provides $z_m = e - 1 \simeq 1.72$ in the limit $\alpha \rightarrow 1$. Such a value is similar to the one predicted by an open FRW model with the matter density parameter of the order of $\Omega_m \sim 0.4$ (see Table 1 of [15]) as well as to the one predicted by a flat model with cosmological constant and $\Omega_m \simeq 0.2$ (see Table 1 of [18]). In Fig. 1 we show the

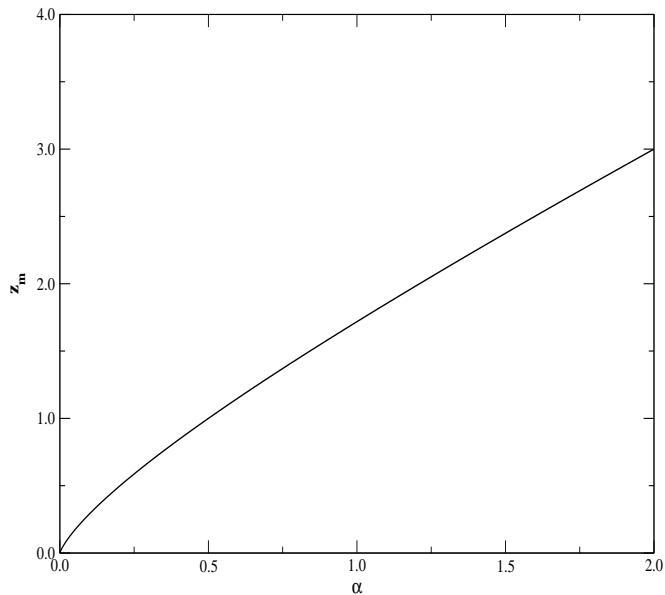


FIG. 1: Critical redshift z_m as a function of the dimensionless parameter α . As expected, for $\alpha = 2/3$ the standard prediction, $z_m = 5/4$, is recovered. For $\alpha \rightarrow 1$, the critical redshift approaches 1.72.

diagram z_m as a function of the dimensionless parameter α . Clearly, from the above equation, z_m is an increasing function of α . Note still that for values of $\alpha \lesssim 0.5$ the minimal redshift takes small values ($z_m \lesssim 1$), which is not observed from the current data. As expected, for $\alpha = 2/3$ the standard prediction, $z_m = 5/4$, is readily recovered.

IV. CONSTRAINTS FROM HIGH- z ANGULAR SIZE MEASUREMENTS

In this section we study the constraints on the parameter α from the angular size measurements of high- z milliarcsecond radio sources. To place such constraints we use the $\theta(z)$ data compiled by Gurvits *et al.* [19]. This data set, originally composed by 330 sources distributed over a wide range of redshift ($0.011 \leq z \leq 4.72$), was reduced to 145 sources with spectral index between $[-0.38, 0.18]$ and total luminosity $Lh^2 \geq 10^{26}$ W/Hz in order to minimize any possible dependence of the angular size on the spectral index and/or linear size on luminosity [19]. This new subsample was distributed into 12 bins with 12-13 sources per bin. In our analysis we assume that possible evolutionary effects can be removed from this sample since compact radio jets are (i) typically less than 100 pc in extent, and therefore, their morphology and kinematics do not depend considerably on the intergalactic medium and (ii) they have ages of the order of years, i.e., much smaller than the cosmological scale of time H_0^{-1} [20]. This particular data set has been exten-

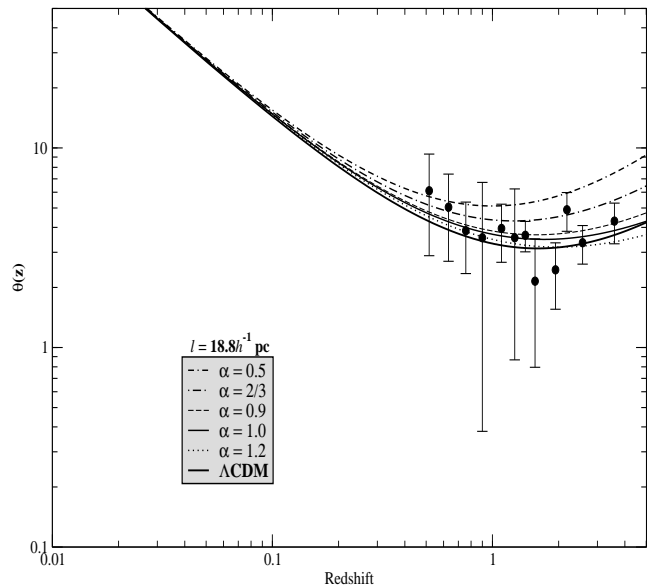


FIG. 2: Angular size versus redshift for selected values of α . The data points correspond to 145 sources binned into 12 bins [19]. In all curves the characteristic angular size has been fixed at $l = 18.8h^{-1}$ pc ($D = 1.28$ mas). The thick line is the Λ CDM prediction.

sively used in the recent literature, with several authors aiming mainly at constraining different quintessence scenarios [21].

To determine the confidence regions in the plane $D - \alpha$ we use a χ^2 minimization for the range of α and D spanning the interval $[0,1]$ and $[0.01,2]$, respectively

$$\chi^2 = \sum_{i=1}^{12} \frac{[\theta(z_i, D, \alpha) - \theta_{oi}]^2}{\sigma_i^2}, \quad (9)$$

where $\theta(z_i, D, \alpha)$ is given by Eq. (7) and θ_{oi} stands for the observed values of the angular size with errors σ_i of the i^{th} bin in the sample.

Figure 2 shows a log-log plot of the angular size versus redshift for power-law cosmologies with some selected values of the dimensionless parameter α . For the sake of comparison, the current favoured cosmological model, namely, a flat scenario with $\sim 70\%$ of the critical energy density dominated by a cosmological constant (Λ CDM) is also shown. In Fig. 3 we show the contours of constant likelihood (68% and 95%) in the plane $\alpha - D$. Note that the data set allows a large interval for the parameter α , which shows the impossibility of placing very restrictive constraints on the index α from the current $\theta(z)$ data. From this analysis we obtain $\alpha = 1.006$ and $D \simeq 1.28$ ($l \simeq 18.8h^{-1}$ pc) as the best fit values. This particular value of α is very close to the one obtained in Sec. II by using recent age estimates of globular clusters and also supports the idea of a strictly linear evolution of $R(t)$ (i.e., $R(t) \propto t$). In Fig. 4, by marginalising over the

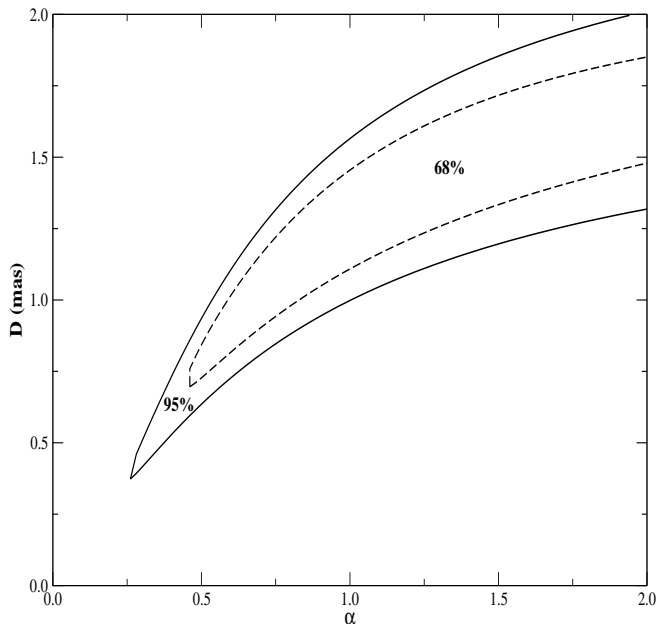


FIG. 3: Confidence regions in the $\alpha - D$ plane according to the updated sample of angular size data of Gurvits *et al.* [19]. The contours correspond to 68% (dashed) and 95% (solid) confidence levels.

characteristic angular scale D , we show the normalised likelihood curve ($\mathcal{L}/\mathcal{L}_{max}$) as a function of the parameter α . The best-fit value for this parameter corresponds to the maximum value of the likelihood function

$$\mathcal{L}(\alpha) = \int \exp(-\chi^2(\alpha, D)) dD. \quad (10)$$

As can be seen from that figure, the likelihood is a sharply function of α with its maximum value centered at $\alpha = 1.006$. We also obtain $0.704 \leq \alpha \leq 1.312$ within 68% c.l. and $0.428 \leq \alpha \leq 1.908$ within 95% c.l..

At this point, it is also interesting to compare our results with some other independent constraints on the power-law index α . For example, Dev *et al.* [9] used SNe Ia data to show that a good agreement between these data and the class of power-law cosmologies studied here is possible for values of $\alpha = 1.0 \pm 0.04$. For a open scenario a study of the statistical properties of gravitational lenses provided $\alpha = 1.13^{+0.4}_{-0.3}$ at 68% c.l. while the predicted number of lensed quasars for the same sample implied $\alpha = 1.09 \pm 0.3$ [10]. Age estimates of high- z galaxies require a lower limit of $\alpha \geq 0.8$ [10]. All these determinations agree satisfactorily with the limits obtained from the analysis presented in this paper.

V. CONCLUSION

We have studied some properties of the angular size - redshift relation in a general class of power-law cosmolo-

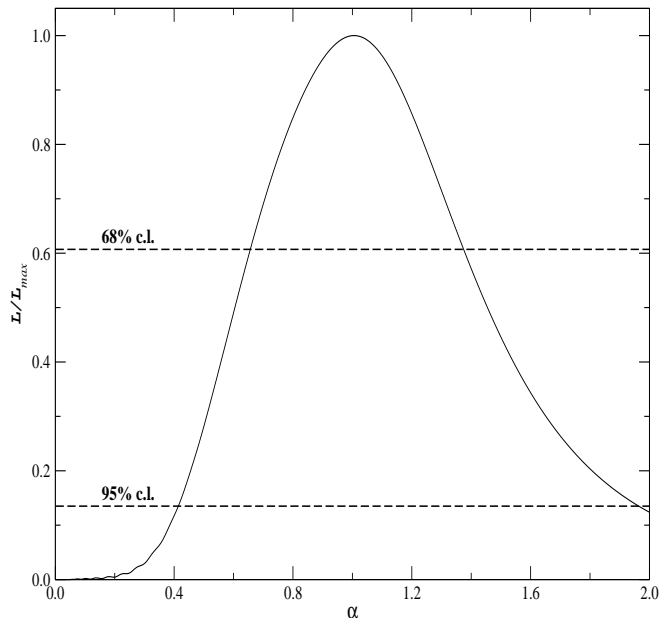


FIG. 4: The likelihood curve, normalised with the maximum likelihood value, is shown as a function of the index α . The remaining parameter D (or equivalently, l) has been marginalised. As indicated, the horizontal dashed lines correspond to confidence limits of 68% and 95%.

gies in which the scale factor is expressed as a function of an arbitrary dimensionless parameter α . We have investigated the influence of this parameter on the redshift z_m at which the angular size takes its minimal value. Moreover, by using measurements of the angular size for a large sample of milliarcsecond compact radio sources we also have placed new constraints on the power-law index α . In particular, after marginalising over the characteristic angular scale D we have found $\alpha = 1 \pm 0.3$ at 68% c.l., a result that agrees very well with other independent determinations based on different methods. We emphasize that if a definitive agreement between the constraints derived from classical cosmological tests and primordial nucleosynthesis could be shown, this class of power-law cosmologies would constitute an interesting alternative to the standard cosmology.

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